Bilateral Exposures and Systemic Solvency Risk

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1. Introduction : two features of financial risks

Diversification & Non Linearities

Diversification may diminish but cannot eliminate risk.

Due to the existence of common risk factors (called systematic risk factors) such as : longevity risk, business cycle, prime rate affecting all adjustable rate mortgages (ARM)...
These factors introduce a dependence between risks.

Nonlinearities drive the financial world.

Nonlinearities can be due to : the design of derivatives (call option), the banking regulation (provision rules), the individual events, such as default, prepayment, lapse (for life insurance)...
Some nonlinearities are hidden in the balance sheet of the banks and insurance companies as seen from Merton's model (Value-of-the-Firm model).

Value-of-the-Firm model

- Stylized balance sheet : Asset (A), Liability (L) and value (Y)
- Bondholders' interest : L^* nominal value of the debt
- Shareholders' interest (with limited liabilities) : initial equities transformed into the net value of assets over liabilities

	Bondholders	Shareholders
$L^* < A$	L^*	$A - L^*$
$L^* > A$	A	0
	$L = min(A; L^*)$	$Y = (A - L^*)^+$

NB : $Y = (A - L^*)^+$ is equivalent to $Y = (A - L)^+$.

Implications

In nonlinear systems, small shocks can have a major impact.

Examples :

- A small shock can make a risky interest rate lower than a risk-free interest rate ; then perfect arbitrage opportunities appear, amplified by leverage effects.
- A switch of the correlation sign between two risks turns a diversified portfolio into a risky portfolio.

2. Balance sheet and exposure

Two approaches in academic literature

- A reduced form based on market returns of the banks and insurance companies, with descriptive measures of systemic risk :
  - CoVaR [Adrian, Brunnermeier (2010)]
  - Marginal Expected Shortfall [Acharya et alii (2010), Brownlees, Engle (2010), Acharya, Engle, Richardson (2012)]
- A structural approach considering the risk exposure hidden in the balance sheets of banks and insurance companies. A lack of data, which explains why this part of the literature focused on the clearing systems and the short term interbank lending [Upper, Worms (2004), Eisenberg, Noe (2001)].

Since the exposure data are collected by the regulators for financial stability (and also independently for hedge funds), the structural approach will likely be largely applied rather soon.

We give an example of such a structural implementation.

Perimeter of a system

The perimeter is defined with respect to :

- Institutions : **banks**, insurance companies, hedge funds...
- Consolidation : **banking group**, with/without off-balance sheet...
- Currency : **Euro** (after conversion), Dollars...
- Financial contagion channel : **stocks**, **lending+loans**, derivatives, mutualization features...

However, one may understand a banking system as :

i) **The set of banks**

ii) A virtual bank obtained by consolidating all the existing banks (see e.g. BIS data)

Balance sheet of a bank i

- Liability : all type of bonds (or loans) for a nominal value L_i^*
- Asset :
  - Cross-participation : $\pi_{i,j}$ is the proportion of the value of bank j owned by bank i
  - Interbank lending : $\gamma_{i,j}$ is the proportion of total lending granted to bank j owned by bank i
  - Other assets : Ax_i gathers the assets of all other counterparties : depositors, retail, corporate, banks and insurance companies out of the perimeter...
- Value of bank i : Y_i

Balance sheet of a bank i

Asset	Liability
$\pi_{i,1} Y_1$	L_i
$\vdots$	
$\pi_{i,n} Y_n$	
$\gamma_{i,1} L_1$	
$\vdots$	
$\gamma_{i,n} L_n$	
Ax_i	
A_i	L_i

Matrices of exposures

- Cross-participation :

$$\Pi = \begin{pmatrix} \pi_{1;1} & \cdots & \pi_{1;n} \\ \vdots & & \vdots \\ \pi_{n;1} & \cdots & \pi_{n;n} \end{pmatrix}$$

- Interbank lending :

$$\Gamma = \begin{pmatrix} \gamma_{1;1} & \cdots & \gamma_{1;n} \\ \vdots & & \vdots \\ \gamma_{n;1} & \cdots & \gamma_{n;n} \end{pmatrix}$$

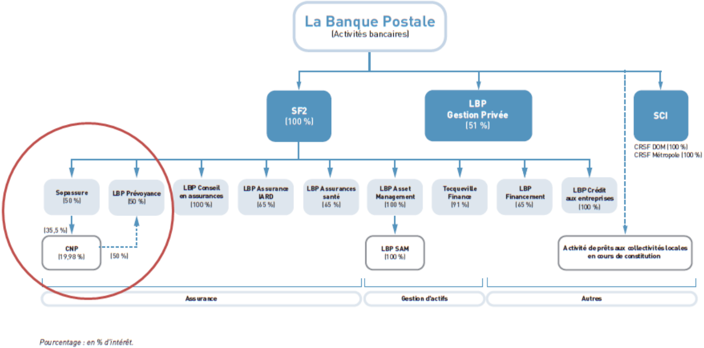
Perimeter

In the paper, we consider a network of 5 large French banking groups. This set accounts for :

- the various business model of French groups, namely mutual banks
- some local/international activity

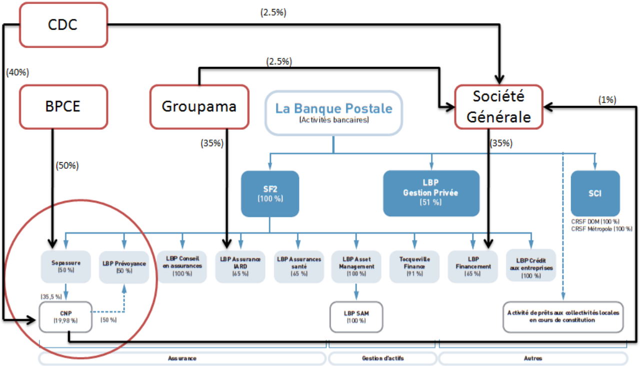
We keep 5 banks to have reasonable size of vectors and matrices. We used the public financial statements of these banks to estimate an illustrative network.

Example of interconnections between banks and insurance



Source: public financial statements of Groupe Laposte

Example of interconnections between banks and insurance



Source: public financial statements of Groupe Laposte, SG, Groupama, CDC, CNP.

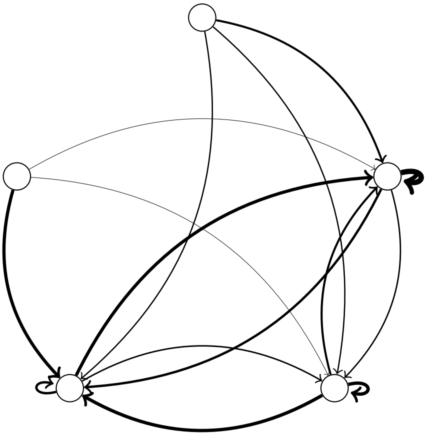
Exposure Matrices

At 12/31/2012, exposure matrix:

$$\Pi = \begin{pmatrix} 0 & 0 & 0 & \mathbf{2.5} & 0.19 & \mathbf{40} & 0 & 0.3 \\ 0 & 0 & 0 & 0.11 & 0.23 & \mathbf{17.8} & 0 & 0.31 \\ 0 & 0 & 0 & 0.34 & 0.18 & \mathbf{17.8} & 0 & 0.57 \\ 0 & 0 & \mathbf{2} & \mathbf{4} & 0.32 & 0.01 & 0 & 0.82 \\ 0 & 0 & 0 & 0.27 & \mathbf{3} & 0.15 & 0 & \mathbf{5} \\ 0 & 0 & \mathbf{2} & \mathbf{1} & 0.12 & 0.89 & 0 & 0.35 \\ 0 & 0 & \mathbf{2} & \mathbf{2.5} & 0.01 & 0.12 & 0 & 0.45 \\ 0 & 0 & 0 & 0.05 & \mathbf{5} & 0.56 & 0 & 0 \end{pmatrix}$$

Source: public financial statements

Network from Π



3. Liquidation equilibrium

Equilibrium conditions

Assumptions :

- n banks
- the portfolios are crystallized : Π, Γ and L^* are fixed.

The liquidation equilibrium is defined by the $2n$-dimensional system :

$$\begin{cases} Y_i = \left[\sum_{j=1}^{n} (\pi_{i,j} Y_j) + \sum_{j=1}^{n} (\gamma_{i,j} L_j) + Ax_i - L_i \right]^+ , \\ L_i = \min \left[\sum_{j=1}^{n} (\pi_{i,j} Y_j) + \sum_{j=1}^{n} (\gamma_{i,j} L_j) + Ax_i \quad , \quad L_i^* \right], \end{cases}$$

for $i = 1, ..., n$.

Equilibrium solution

Proposition 1 : If $\pi_{i,j} \geq 0$, $\gamma_{i,j} \geq 0$, $\forall i,j$, $\sum_i \pi_{i,j} < 1$, $\forall j$, $\sum_i \gamma_{i,j} < 1$, $\forall j$, the liquidation equilibrium (Y, L) exists and is unique for any choice of Ax and L^*.

The "inputs" (shocks) are the Ax_i and the equilibrium concerns both the values of the firms and the values of the debts : L_i, Y_i, for $i = 1, ..., n$.
This system is nonlinear due to threshold effects.

Case of two banks

Let us consider the basic network composed of two banks : bank 1 and bank 2.

Bank 1

Asset	Liability
$\pi_{1,1} Y_1$	L_1
$\pi_{1,2} Y_2$	
$\gamma_{1,1} L_1$	
$\gamma_{1,2} L_2$	
Ax_1	
A_1	L_1

Bank 2

Asset	Liability
$\pi_{2,1} Y_1$	L_2
$\pi_{2,2} Y_2$	
$\gamma_{2,1} L_1$	
$\gamma_{2,2} L_2$	
Ax_2	
A_2	L_2

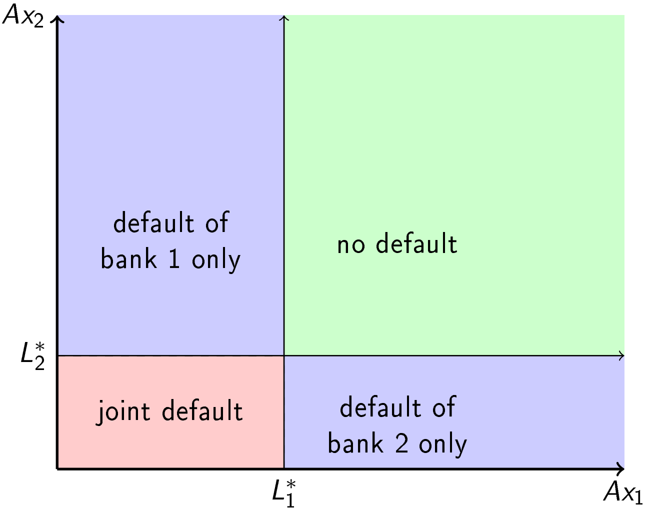
Case of two banks

Each bank can be either alive, or in default. Therefore, there are $2^2 = 4$ possible regimes :

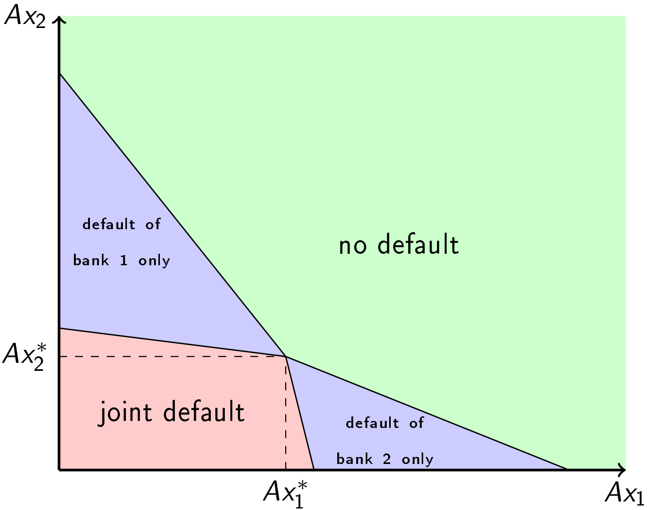
- Regime 1 : both bank 1 and bank 2 are alive
- Regime 2 : both bank 1 and bank 2 default
- Regime 3 : bank 1 defaults while bank 2 is alive
- Regime 4 : bank 1 is alive while bank 2 defaults

The previous proposition states that, under portfolio crystallization, only one of the four previous regimes can arise.

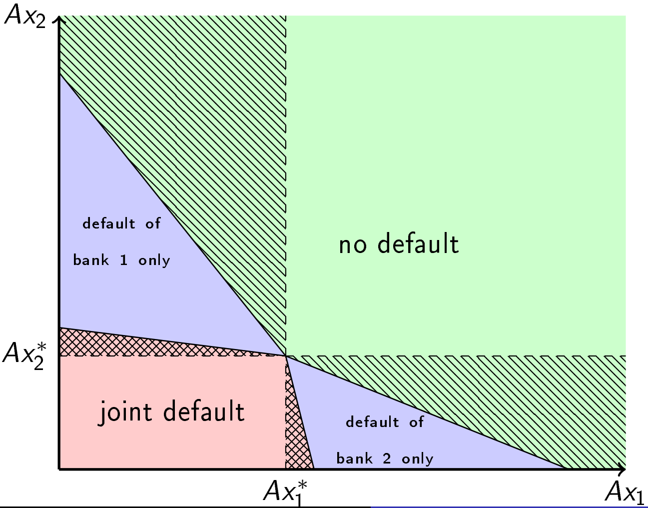
Regimes of default **without** interconnections



Regimes of default **with** interconnections



Regimes of default **with/out** interconnections



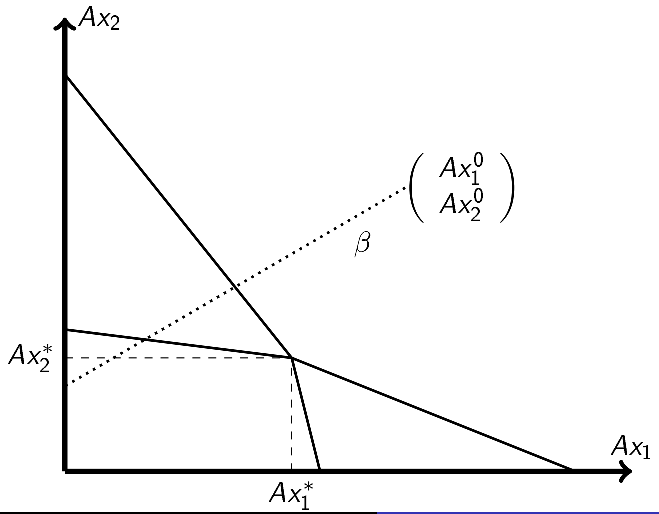
Impulse response analysis

An exogenous shock affects the asset out of the banking network, Ax.
Let us consider a linear shock of magnitude δ and direction β that affect the initial exogenous assets Ax^0 :

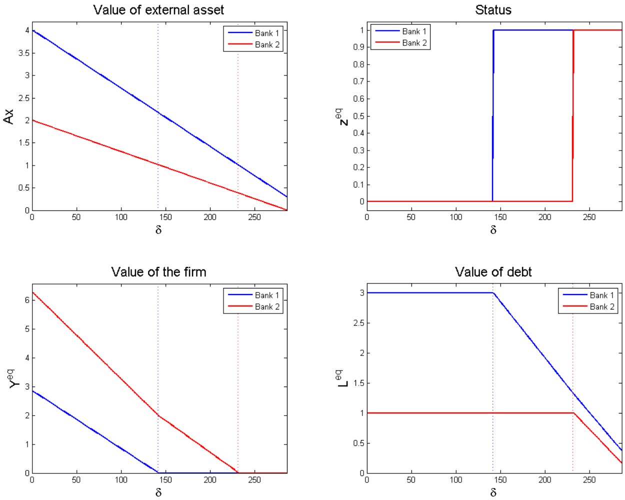
$$Ax = Ax^0 + \delta\beta.$$

For given β, the equilibrium can be computed for any value of δ (whenever Ax is positive).
β can be seen as a factor of loss in a fixed scenario (GDP, market index, or individual loss) and δ as the severity of the scenario.

A common adverse deterministic shock



A common adverse deterministic shock



Stochastic shock

Alternatively, we can consider Ax as stochastic. The regimes defined in Proposition 1 are characterized by sets defined on Ax. Therefore the multidimensional distribution of Ax can be used to compute the probability that each regime arises. Equivalently, we get the probability of default (PD) of a given bank, or of a set of banks.

Explicit formulas can be very complicated, but simulations can easily be carried out.

4. Contagion measure

Identifying the contagion effect

Let us consider a shock of magnitude δ and direction β affecting external assets : $Ax = Ax^0 + \delta\beta$.

By Proposition 1, we can define the values of the firms and the values of debts at equilibrium as functions of the shock (δ, β) and the balance sheet characteristics $\mathcal{S}^0 = \{\Pi, \Gamma, L^*, Ax^0\}$:

$$\begin{cases} Y(\mathcal{S}^0; \delta, \beta) \\ \\ L(\mathcal{S}^0; \delta, \beta) \end{cases}$$

Identifying the contagion effect

Let us assume that in the initial situation the banks transform their crossholding into cash. We get a new balance sheet with external asset components equal to :

$$\widetilde{Ax_i^0} = \Pi_i Y^0 + \Gamma_i L^* + Ax_i^0,$$

and zero interconnections :

$$\Pi = \Gamma = 0.$$

The system becomes :

$$\widetilde{\mathcal{S}^0} = \{0, 0, L^*, \widetilde{Ax_i^0}\}.$$

Finally, we compute the associated equilibrium values :

$$\begin{cases} Y(\widetilde{\mathcal{S}^0}; \delta, \beta), \\ L(\widetilde{\mathcal{S}^0}; \delta, \beta). \end{cases}$$

Basic statistics

One can use basic statistics to compare the two situations :

i) The number of non-defaulted banks :

$$N_0 = \sum_{i=1}^{n} \mathbb{1}_{Y_i > 0} = \sum_{i=1}^{n} \mathbb{1}_{L_i - L_i^* = 0},$$

where $\mathbb{1}_A$ denotes the indicator function of A.

ii) The total value of the banks :

$$\bar{Y} = \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} Y_i \mathbb{1}_{Y_i > 0};$$

iii) The total value of the debt :

$$\bar{L} = \sum_{i=1}^{n} L_i = \sum_{i=1}^{n} L_i^* \mathbb{1}_{Y_i > 0} + \sum_{i=1}^{n} L_i \mathbb{1}_{L_i < L_i^*}.$$

Direct effect and contagion effect

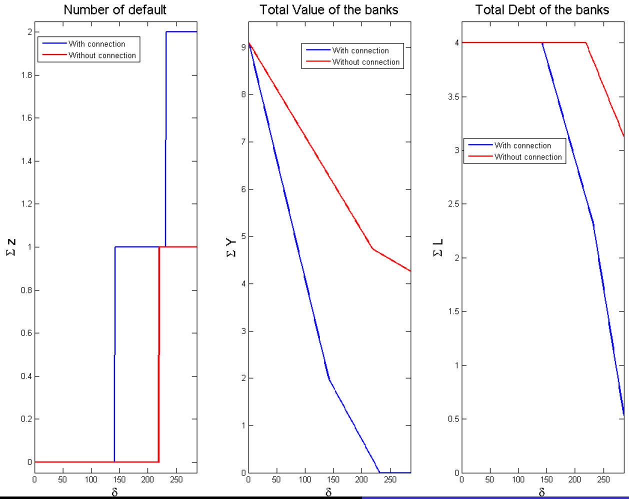
For instance, let us consider the number of non-defaulted banks computed in the two situations :

$$N_0(\mathcal{S}_0) \quad \text{and} \quad N_0(\widetilde{\mathcal{S}_0}).$$

The decomposition between direct effect and contagion is :

$$N_0(\mathcal{S}_0) = \underbrace{N_0(\widetilde{\mathcal{S}_0})}_{\text{Direct Effect}} + \underbrace{\left(N_0(\mathcal{S}_0) - N_0(\widetilde{\mathcal{S}_0})\right)}_{\text{Contagion Effect}}.$$

Direct effect and contagion effect



Reverse Stress-Tests

In our framework, a reverse stress-test exercise looks for the smallest magnitude δ^* of shock that triggers a specific event, say the first default. Let us consider a shock specific to bank i :

$$Ax = Ax^0 - \delta(0, ..., 0, Ax_i, 0, ...0),$$

and compute the δ^*'s with and without contagion. The difference between the two gives an insight of the effect of the interconnections on bank's resilience.

Reverse Stress-Tests

Specific shock on	A	B	C	D	E
First bank to fail	A	B	C	D	E
δ (with contagion, %)	5.810	4.544	3.073	4.559	4.340
δ (without contagion, %)	5.810	4.544	3.085	4.635	4.353
$1 - L_i^*/Ax_i^0$ (%)	1.77	-0.98	-5.36	1.76	3.19

Table: Reverse Stress-Tests for the Banking Sector (at 12/31/2010) ; δ in percent

Based on this perimeter and on this shock, we illustrate that :

- two banks are not affected by the interconnections,
- generally speaking, interconnections has a positive impact.

Stochastic shocks

The decomposition can also be used with stochastic shocks.
It is difficult to compare the whole distribution of the values of the firms and of the debts in the two situations. However, we can focus on some summary statistics. An appealing one in the framework of Basel regulation is the individual probability of default (PD).

Stochastic shocks : a simple model

We introduce stochastic shocks on the exogenous asset components as in the standard Vasicek extension of the Value-of-the-Firm model :

$$log(Ax_i) = log(Ax_i^0) + u_i,$$

$$u_i \sim \mathcal{N}\left(0; \sigma^2 Id\right), \quad i = 1, ..., n.$$

The *PD* with and without connection can be estimated by simulations.
Let us consider the French banking sector with independent Gaussian shocks ($\sigma = 0.0141$).

Stochastic shocks : a simple model

	PD (in %) Without connection	PD (in %) With connection	ΔPD
A	0.001	0.000	-0.001
B	0.056	0.025	-0.003
C	1.348	1.391	+0.043
D	0.052	0.001	-0.041
E	0.091	0.002	-0.089

Table: Simulated Probabilities of Default for the Banking Sector (at 12/31/2010) ; 100,000 simulations

Being interconnected lowers the probability of default. The interconnections can be seen as an efficient diversification of risk since the stochastic shocks u_i's, are independent in our exercise.

5. Conclusion

Conclusion

Contagion phenomena are analyzed by a structural model based on the balance sheets of the financial institutions.
This framework is appropriate for stress-test exercises with "second round" effects, based on either deterministic, or deterministic shocks.
Moreover, the model includes the possibility to disentangle the direct impact of a shock from the contagion effect.

Liquidation Equilibrium with Seniority and Hidden CDO

The analysis can be extended to account for different levels of seniority of the debt :
Gouriéroux, Héam, Monfort : "Liquidation Equilibrium with Seniority and Hidden CDO"
This allows for a careful analysis of the prices of the junior and senior tranches written on a single bank, that is, on a single name. Since the balance sheet of the bank include junior and senior debts of other institutions, these tranches are in fact written on a portfolio of junior and senior debts, that is, are written on several names : "hidden CDO".

Liquidation Equilibrium with Seniority and Hidden CDO

The price of this hidden CDO has to account for the joint defaults and recovery rates at liquidation equilibrium.

price of a tranche written on a single name

=

price of a hidden CDO

=

"standard price of such a CDO"

+

"price of contagion"